19/1/22

B.Sc. Part I (Hons)

1st Paper — Matrices

Rank of a matrix A

~~Let r be the~~

Let A be a given matrix of order $m \times n$.

Let r be a natural number, such that $r < m$ and $r < n$.

Then r is the rank of the matrix if

(a) there is at least one minor of order ~~r~~ r which do not vanish,

(b) all the minors of order $(r+1)$ vanish.

Examples

1. Let $A = \begin{bmatrix} 1 & 3 & 4 & 7 \\ 2 & 4 & 5 & 8 \\ 3 & 1 & 2 & 3 \end{bmatrix}$

$\Rightarrow$ There is no minor of order 4.

Now, minor of order 3

$$= \begin{vmatrix} 1 & 3 & 4 \\ 2 & 4 & 5 \\ 3 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 4 \\ 2 & 4 & 5 \\ 0 & -6 & -7 \end{vmatrix}$$

$[R_3 \to R_3 - (R_1 + R_2)]$

$$= - \begin{vmatrix} 1 & 3 & 4 \\ 2 & 4 & 5 \\ 0 & 6 & 7 \end{vmatrix} = -[2 - 3(14) + 4(12)] \neq 0$$

So, there is at least one minor of order 3 which does not vanish.

$\Rightarrow$ rank of $A = 3$.

2. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 7 & 8 \\ 4 & 9 & 11 \end{bmatrix}$.

Here, the only minor of order 3 $= \begin{vmatrix} 1 & 2 & 3 \\ 3 & 7 & 8 \\ 4 & 9 & 11 \end{vmatrix}$

$= \begin{vmatrix} 1 & 2 & 3 \\ 3 & 7 & 8 \\ 0 & 0 & 0 \end{vmatrix}$ $\quad R_3 \to R_3 - (R_1 + R_2)$

$= 0$.

$\Rightarrow$ All minors of order 3 vanish

$\Rightarrow$ rank of A is not 3 but less than 3.

Now, minor of order 2 $= \begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix} = 7 - 6 = 1 \neq 0$

So, there exists at least one minor of order 2 which do not vanish.

$\Rightarrow$ rank of $A = 2$

i.e. $\rho(A) = 2$.